Scattering cross-section :-

A beam of incident particle is characterized by intensity (I) or flux.

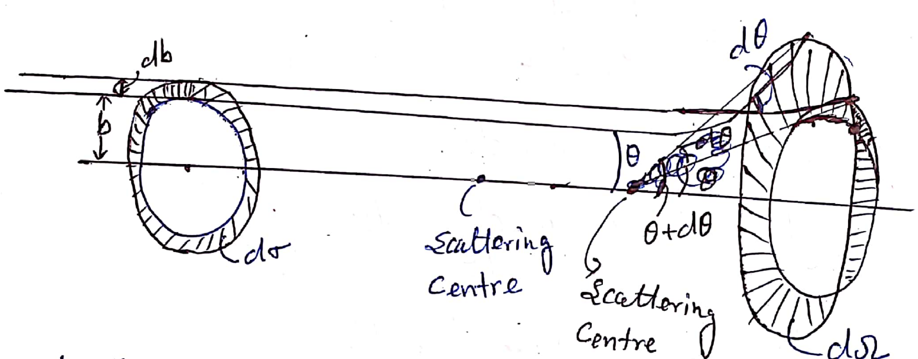
It is define as the number of particles of the beam passing per unit area normal to the beam per unit time.

Suppose dN represent the number of particles in the beam scattered per unit time through scattering angle lying between θ and $\theta + d\theta$.

Let $d\sigma$ is the effective cross-section is define as

$$d\sigma = \frac{dN}{I} \quad (cm^2) \quad \text{—} (1)$$

(∵ No. of particle per unit time $= \frac{dN}{t}$)



'b' represent the impact parameter which is perpendicular distance between the centre of force field and incident beam velocity vector.

Now the number of scattered particle per unit time

$$dN = dA \times I$$

$$dN = 2\pi b\, db\, I$$

($dA = 2\pi b db$; $dA = 2\pi r dr$)

$$d\sigma = \frac{dN}{I}$$

$$d\sigma = \frac{2\pi b\, db\, \cancel{I}}{\cancel{I}}$$

$$d\sigma = 2\pi b\, db \quad \text{--- (11)}$$

Let us chose $d\Omega$ is the solid angle lying between angular range θ and $\theta + d\theta$.

Now whole solid angle

$$\int d\Omega = \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$\int d\Omega = -\cos\theta \Big|_0^{\pi} \left(\phi \right)_0^{2\pi}$$

$$\int d\Omega = 2 \times 2\pi$$

$$\boxed{\int d\Omega = 4\pi}$$

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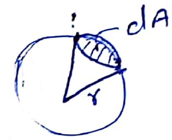
$$d\Omega = \frac{dA}{r^2}$$



$$d\Omega = 2\pi \sin\theta\, d\theta$$

$\frac{d\sigma}{d\Omega}$ is define as differential scattering cross-section

$$\sigma_T = \int \frac{d\sigma}{d\Omega}\, d\Omega$$

$$\because d\sigma = 2\pi b\, db$$

$$d\sigma = 2\pi b \frac{db(\theta)}{d\theta} d\theta$$

$$\frac{d\sigma}{d\Omega} = \frac{\cancel{2\pi} b \frac{db}{d\theta} d\theta}{\cancel{2\pi} \sin\theta\, d\theta}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{b(\theta)}{\sin\theta} \left| \frac{db}{d\theta} \right|}$$